

# A Fast Radial Basis Functions Method for Solving Partial Differential Equations on Arbitrary Surfaces <sup>†</sup>

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**Abstract:** Many applications in the natural and applied sciences involve the solution of partial differential equations (PDEs) on surfaces. Application areas for PDEs on static surfaces include image processing, biology, and computer graphics. Applications for PDEs on moving surfaces also occur frequently. Notable examples arise in biology, material science, fluid dynamics, and computer graphics. There are three main categories of methods for solving PDEs on arbitrary surfaces: the methods that rely (1) on parametrization, (2) on an embedding, and (3) on triangulation. Parametrization-type methods necessitate the discretization of the parametrized differential operators, which can be quite challenging. With embedding-type methods, it is possible to bypass this obstacle and instead use the standard  $R^3$  operators by embedding the surface in the ambient space,  $R^3$ . One of the most common embedding methods is the closest point method (CPM). The surface is enclosed inside a thick layer of nodes that belong to a dense three-dimensional grid. Each one of these nodes takes the function value of the one associated with their closest point to the surface, implicitly imposing that the normal derivatives at each node are null. Under that constraint, the surface Laplacian (for instance) is equivalent to its  $R^3$  analog. Another embedding method called the Radial Basis Functions Orthogonal Gradients method (RBF-OG<sub>r</sub>) was introduced in. This method is different as every computation is performed on the surface, and the null normal derivative constraints are imposed differently. We take advantage of the meshfree character of RBFs, which give the flexibility to represent complex geometries in any spatial dimension while providing a high order of accuracy. Both the RBF-based CPM [4] and the RBF-OG<sub>r</sub> methods show much promise in the solution of PDEs on both static and evolving smooth surfaces. We will present several examples to illustrate the numerical convergence properties of our proposed methods.

**Keywords:** RBF-OG<sub>r</sub> method; radial basis function; partial differential equation.

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## Conflicts of Interest

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